Cosmological Hysteresis and the Cyclic Universe

Varun Sahni[a] and Aleksey Toporensky[b]

[a] *Inter-University Centre for Astronomy and Astrophysics,*

Post Bag 4, Ganeshkhind, Pune 411 007, India and

[b]*Sternberg Astronomical Institute, Moscow State University,*

Universitetsky Prospekt, 13, Moscow 119992, Russia

A Universe filled with a homogeneous scalar field exhibits '*Cosmological hysteresis*'. Cosmological hysteresis is caused by the asymmetry in the equation of state during expansion and contraction. This asymmetry results in the formation of a *hysteresis loop*: $\oint pdV$, whose value can be non-vanishing during each oscillatory cycle. For flat potentials, a negative value of $\oint pdV$ leads to the *increase* in amplitude of consecutive cycles and to a universe with older and larger successive cycles. Such a universe appears to possess an *arrow of time* even though entropy production is absent and all of the equations respect time-reversal symmetry ! Cosmological hysteresis appears to be widespread and exists for a large class of scalar field potentials and mechanisms for making the universe bounce. For steep potentials, the value of $\oint pdV$ can be positive as well as negative. The expansion factor in this case displays quasi-periodic behaviour in which successive cycles can be both larger as well as smaller than previous ones. This quasi-regular pattern resembles the phenomenon of *beats* displayed by acoustic systems. Remarkably, the expression relating the increase/decrease in oscillatory cycles to the quantum of hysteresis appears to be *model independent*. The cyclic scenario is extended to spatially anisotropic models and it is shown that the anisotropy density decreases during successive cycles if $\oint pdV$ is negative.

1. INTRODUCTION

We live in a universe that is old and very nearly spatially flat. The possibility that these two properties of our universe – its age and small spatial curvature – could be related, has

been the focus of considerable study in cosmology. Tolman wondered whether a progressively older universe could be constructed out of repeated cycles of expansion and contraction [1]. However he was also well aware of the fact that, for perfect fluids, the equations of motion are reversible and so each cyclic epoch is identical to the next. To construct a universe in which successive epochs were of longer duration Tolman postulated the presence of a viscous fluid. Viscosity leads to an asymmetry in pressure during expansion and contraction which, in turn, results in a progression of cyclic epochs of successively longer duration.

However Tolman did not have a prescription for avoiding the Big Bang singularity and had to assume it a priori. An important later development which addressed both the age issue and the flatness problem was inflation. By driving the flatness parameter, Ω, towards unity, inflation ensured that the rapidly expanding universe, even if spatially closed, would expand for a long duration of time. However inflation did not address the issue of the big bang singularity and it has been shown that although inflation could be eternal in the future, its past spacetime is necessarily incomplete [2].

The present paper further develops the central idea's of an oscillatory universe and attempts to synthesise elements of cyclic cosmology with the inflationary paradigm. Extending the arguments originally proposed in [3] we demonstrate that a universe filled with a scalar field possesses the intriguing property of '*hysteresis*'. Cosmological hysteresis is related to the fact that the pressure of a scalar field is usually asymmetric with respect to expansion and contraction: $P_{\rm expansion} < P_{\rm contraction}$. This asymmetry leads to the development of a *hysteresis loop*, $\oint PdV \neq 0$, during each oscillatory cycle. The loop can cause consecutive cycles to be larger in amplitude and in duration. While the asymmetry between expansion and contraction is largest for inflationary potentials, the phenomenon of cosmological hysteresis appears to be *generic* and is observed also in potentials which do not give rise to inflation.

In § 2 of this paper we develop the equations which relate cosmological hysteresis to the amplitude of successive cycles and demonstrate that these equations have a universal form which is independent of the scalar field potential responsible for hysteresis. Furthermore, the presence of hysteresis appears to be robust, and is shown to exist for quite general mechanisms of singularity avoidance such as those predicted by Braneworld cosmology [4] and Loop Quantum Gravity [5, 6]. In § 3 we demonstrate that $\oint PdV < 0$ for flat potentials, which leads to the increase in amplitude of successive cycles. A remarkable feature of this scenario is that the universe appears to possess an *'arrow of time'* even though the field

equations are formally time reversible ! For steep potentials the value of $\oint PdV$ can be negative as well as positive. In this case the phenomenon of Cosmological hysteresis can adorn the universe with quasi-regular oscillations, or *beats*, resembling those in acoustic systems. Section 4 discusses the behaviour of a massive scalar field during cosmological contraction. One finds that the field can grow to sufficiently large values during the contracting phase to give rise to a long duration inflationary phase at the commencement of the next expansion cycle. The behaviour of spatial anisotropy in the cyclic scenario is examined in § 5 and a brief discussion of our results is presented in § 6.

2. COSMOLOGICAL HYSTERESIS

An oscillatory universe requires two essential ingredients: [A] A mechanism for singularity avoidance (when the matter density is high) and [B] a mechanism for inducing contraction (when the matter density is low). Below we provide a brief summary of the assumptions adopted in this paper regarding both A and B.

[A] *Cosmological Bouncing Scenario's* have been widely studied; see [8–13, 17, 18] and [19] for a review. Within the framework of general relativity (GR) a necessary condition for avoiding the big bang singularity is the violation of the energy conditions usually satisfied by matter [20, 21]. An alternative viewpoint considers GR to be an 'effective' theory requiring modification when the space-time curvature becomes enormous. The initial big bang singularity can be successfully replaced with a 'bounce' in theories incorporating both these sets of ideas, examples being Braneworld cosmology [4, 22], Loop-quantum cosmology [5, 6], string theory motivated models [10, 13–16] and other modifications to the Einstein-Hilbert action [23–26].

While a bounce in the early universe could arise in any one of the above scenario's, the central results of this paper will be model independent and so will not depend upon the specific mechanism sourcing the bounce. For illustrative purposes we shall consider a bounce which is known to arise in Braneworld cosmology (with a time-like extra dimension), in which the Friedmann equations are modified to [4]

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\rho\left\{1-\frac{\rho}{\rho_c}\right\}-\frac{k}{a^2}\ , \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left\{(\rho+3p)-\frac{2\rho}{\rho_c}(2\rho+3p)\right\}\ . \end{aligned} \tag{2.1}$$

This equation is also valid in Loop-quantum cosmology (LQC) when $k = 0$ [5, 6]; for spatially open and closed models the LQC equations are discussed in [7]. A more general class of bouncing equations which accomodates (2.1) and FRW dynamics as special cases is ($m \geq 1$)

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\rho\left\{1 - \left(\frac{\rho}{\rho_c}\right)^m\right\} - \frac{k}{a^2} , \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left[(\rho + 3p) - \left\{(3m+1)\rho + 3(m+1)p\right\}\left(\frac{\rho}{\rho_c}\right)^m\right] . \end{aligned} \tag{2.2}$$

Eqn (2.2) reduces to (2.1) when $m = 1$, and to the FRW limit when $\rho_c \to \infty$. In the presence of several components contributing to the pressure and density, one needs to replace $\rho = \sum_i \rho_i$ and $p = \sum_i p_i$ in (2.1) and (2.2).

From (2.2) we see that the universe bounces when $\rho = \rho_c$, at which point $H = 0$ and $\ddot{a} = 4\pi Gm(\rho_c + p_c) > 0$ (where we neglect the curvature term and assume $\rho_c + p_c > 0$). The prevailance of the bounce in (2.1) & (2.2) is not linked to the violation of any of the energy conditions by matter, but is caused instead by a departure of space-time dynamics from the predictions of GR at large values of the matter density.[1] One might also note the following fairly general phenomenological prescription for singularity avoidance [3] which provides a reasonable approximation to (2.1) & (2.2)

$$a \to a, \ \dot{a} \to -\dot{a}, \ \phi \to \phi, \ \dot{\phi} \to \dot{\phi} \ . \tag{2.3}$$

At small values of the density ($\rho \ll \rho_c$) usually associated with late times, higher order terms in the density in (2.1) & (2.2) can be neglected, and cosmic expansion is described by standard FRW equations

$$H^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k}{a^2} , \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i(\rho_i + 3p_i) \ . \tag{2.4}$$

[B] *Cosmological turnaround*, at late times, can take place in several distinct ways:

B1. If the Universe is spatially closed ($k = 1$) and the density of matter drops off faster than a^{-2}. This has been the conventional approach of making a matter/radiation dominated universe turnaround and contract [1, 27]. Indeed for a perfect fluid such

[1] The value of ρ_c is related to fundamental parameters appearing in Braneworld cosmology/Loop quantum cosmology. We do not write them explicitely since the precise form of ρ_c will not be required in this paper.

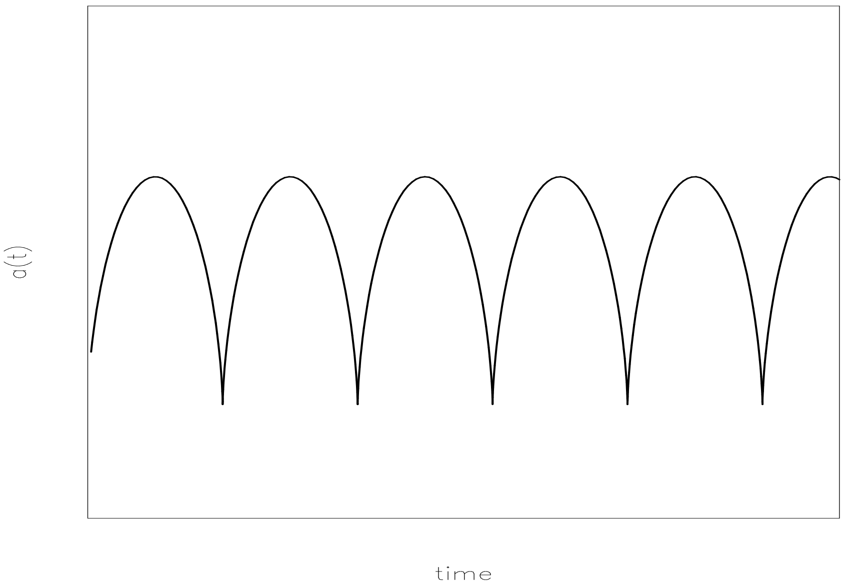

FIG. 1: The expansion factor of a spatially closed matter dominated universe is described by the cycloid (2.5).

as dust ($p = 0$) the expansion factor in a closed universe is time-symmetric and is described by the cycloid (see figure 1)

$$a(\eta) = A(1 - \cos\eta)\,, \qquad t = A(\eta - \sin\eta)\,, \tag{2.5}$$

for which the space-time becomes singular at $a(t) = 0$.

B2. The universe will turnaround if, in addition to 'normal' matter with $\rho(t) \geq 0$, one postulates a form of matter, $\tilde{\rho}$, whose energy density becomes *negative* at late times. Members of this category include:

(i) $\tilde{\rho}(t) = -A/a^n$, with $A > 0$ and $n \leq 2$, where $n = 0$ corresponds to a *negative* cosmological constant $|\Lambda| \equiv A$. (B1 can also be regarded as being a member of this category for $A = k$ and $n = 2$.)

(ii) Scalar fields with potentials $V_1(\phi) \propto \cos\phi$ and $V_2(\phi) = \lambda\phi^4 - m^2\phi^2$, allow $V(\phi)$ to evolve to negative values as the universe expands. These models can therefore source cosmic turnaround. Both potentials contain the possibility of giving rise to a *transiently accelerating* universe thereby providing us with interesting candidates for dark energy [28] as well as cyclic cosmology. Other models of transient acceleration are discussed in [29, 30].

B3. A novel means of using (2.1) to obtain a cyclic universe was suggested in [31]. These authors noted that the density in a phantom dark energy component *increases* as the universe expands, thus making the ρ^2 term in (2.1) relevant both at early and at late times. In this scenario, the bounce at small values of the expansion factor is caused by normal matter, while the universe turns around and contracts due to phantom DE.

The bouncing scenario [A] together with either of [B1]-[B3] gives rise to cyclic cosmology.

In this paper we shall assume that during some period in its history the universe was dominated by a massive scalar field. The presence of a scalar can make cosmological dynamics much more versatile, as we demonstrate below.

In this case the Lagrangian density and the energy-momentum tensor have the form

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\\ T_{ij} &= \partial_i\phi\partial_j\phi - g_{ij}\mathcal{L}\ ,\end{aligned} \tag{2.6}$$

and, for a homogeneous scalar field, the energy density and pressure are, respectively,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,, \tag{2.7}$$

and the scalar field equation of motion is

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0\,. \tag{2.8}$$

The term $3H\dot{\phi}$ in (2.8) behaves like friction and damps the motion of the scalar-field when the universe expands ($H > 0$). By contrast, in a contracting ($H < 0$) universe, $3H\dot{\phi}$ behaves like *anti-friction* and accelerates the motion of the scalar field. Consequently a scalar field with the potential [33] $V = V_0\phi^{2k}$, $k = 1, 2$ displays two asymptotic regimes [34]

$$p \simeq -\rho \ \text{ during expansion } \ (H > 0)\,, \tag{2.9}$$

$$p \simeq \rho \ \text{ during contraction } \ (H < 0)\,. \tag{2.10}$$

In the words of Zeldovich [35] "*There is a moral to be learned from these simple calculations. The result by and large conforms to Braun and Le Chatelier's principle, which also holds in human relations:*

Every system resists outside forces.

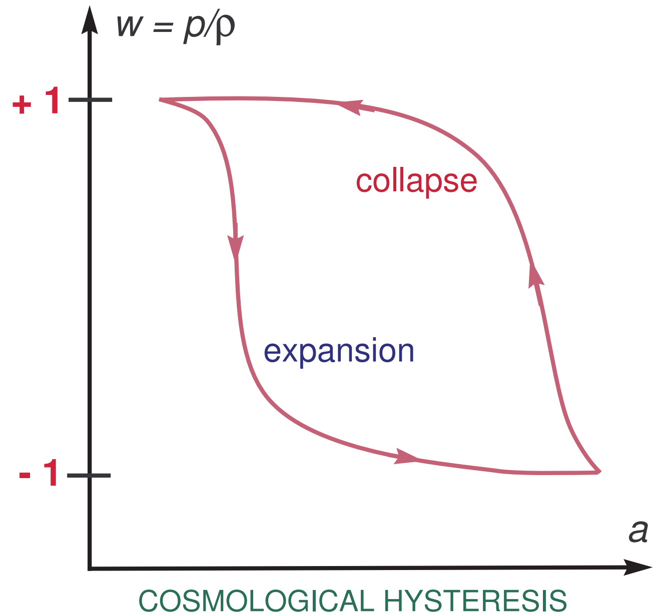

FIG. 2: An idealized illustration of cosmological hysteresis. The *hysteresis loop* shown above has $\oint pdV < 0$ evaluated over a single expansion-contraction cycle. Rapid oscillations of the scalar field will modify this picture, since the equation of state of the scalar field varies between -1 and $+1$ during oscillations. Indeed, as discussed later in the text, scalar field oscillations are indispensible for providing a non-vanishing value to the hysteresis loop. Oscillations of the scalar during expansion scramble its phase making all values of $\dot{\phi}$ equally likely at turnaround and leading to $p_{\rm expansion} \neq p_{\rm contraction}$ and $\oint pdV \neq 0$. By contrast only a unique value of $\dot{\phi}$, namely $\dot{\phi} = 0$ at *turnaround*, enables the scalar field to follow its original trajectory in reverse during contraction, resulting in $p_{\rm expansion} = p_{\rm contraction}$ and $\oint pdV = 0$. Consequently only a very small set of initial conditions does not give rise to hysteresis, the latter appearing to be ubiquitous for scalar field models which can oscillate.

The scalar field expands, and a negative pressure (or tension) builds up. If the expansion were created by the motion of a piston in a cyclinder containing ϕ, the tension would decelerate the piston. On the other hand, if the field is being compressed, a positive pressure builds up, producing a force opposing the motion of the piston."

As Zeldovich suggests, there is nothing really surprising about the behaviour of the scalar field in a closed universe. What is surprising, however, is the observation [3] that as the

universe contracts, the scalar field does not perform a time reversal and move back up the same trajectory down which it descended during expansion. What is seen instead is a lag between the trajectories describing expansion and contraction. This behaviour, shown in figure 2, is typical of systems displaying hysteresis.[2]

Let us now discuss the effect of hysteresis on cosmological dynamics. We assume, as in B2, that the late time behaviour of the universe is governed by the Einstein equation

$$H^2 \simeq \kappa\rho - \frac{A}{a^n}, \tag{2.11}$$

where $\kappa = \frac{8\pi G}{3}$, and $A > 0$, $n \leq 2$, so that the universe turns around and contracts if matter satisfies the strong energy condition $\rho + 3p \geq 0$. Note that $A \equiv \Lambda < 0,\ n = 0$ corresponds to a *negative* cosmological constant, whereas $A = 1,\ n = 2$ describes a universe which is spatially closed.

The scalar field (2.6) has no dissipation and therefore provides us with an example of a perfect fluid [36]. However unlike other perfect fluids such as dust or radiation, the expansion factor for a cyclic universe filled with a scalar field need not display time-symmetric evolution. The reason for this is as follows.

At turnaround the universe stops expanding and begins to contract. Setting $H = 0$ in (2.11) we get

$$\kappa\rho_t = \frac{A}{a_t^n} \tag{2.12}$$

where ρ_t is the density and a_t the expansion factor at turnaround. The mass [3] associated with the volume a^3 is $M = \rho a^3$, therefore at turnaround, $\kappa M_t = A a_t^{3-n}$.

The *work done* during each contraction-expansion cycle is related to the *hysteresis loop*, $\oint pdV$, as follows

$$\delta W = \oint pdV = \int_{\text{contraction}} p\ dV + \int_{\text{expansion}} p\ dV\ . \tag{2.13}$$

Setting $\delta W + \delta M_t = 0$ we get [4]

$$-\oint pdV = \delta M_t = \frac{A}{\kappa}\delta a_t^{3-n}\ , \tag{2.14}$$

[2] Nature is replete with examples of hysteresis. These range from the behaviour of ferromagnets under the influence of an external magnetic field, to control systems, mechanics and even economics !

[3] Our results do not depend upon α in $M = \alpha\rho a^3$ and we set $\alpha = 1$ for simplicity.

[4] The relationship $\delta M = -p\delta V$ follows from the conservation equation $T^k_{i\ ;k} = 0 \Rightarrow \dot\rho + 3H(\rho + p) = 0$. One might note that formulae (2.15) & (2.20) agree with our numerical results for a wide range of parameters.

from where we find the following simple expression relating the *change in amplitude* of successive cycles to the value of the hysteresis loop ($a_{\rm max} \equiv a_t$)

$$\delta \left(a_{\rm max}\right)^{3-n} \equiv \left\{a_{max}^{(i)}\right\}^{3-n} - \left\{a_{max}^{(i-1)}\right\}^{3-n} = -\frac{\kappa}{A} \oint pdV \ , \tag{2.15}$$

where the hysteresis loop is evaluated over one complete *contraction-expansion* cycle, namely

$$\oint pdV := \int_{a_{max}^{(i-1)}}^{a_{max}^{(i)}} pdV \ , \tag{2.16}$$

$a_{max}^{(i)}$ being the maximum value of the expansion factor in the $i^{\rm th}$ cycle; see figure 3.

From (2.15) we find that the change in amplitude of consecutive cycles is sensitive both to the value of the hysteresis loop, $\oint pdV$, and the mechanism responsible for turnaround. Two extreme cases correspond to: (i) the negative cosmological constant ($n = 0$) for which

$$\delta a_{\rm max}^3 = -\frac{\kappa}{\Lambda} \oint pdV \ , \tag{2.17}$$

(ii) the spatially closed universe ($n = 2$) for which

$$\delta a_{\rm max} \equiv a_{max}^{(i)} - a_{max}^{(i-1)} = -\kappa \oint pdV \ . \tag{2.18}$$

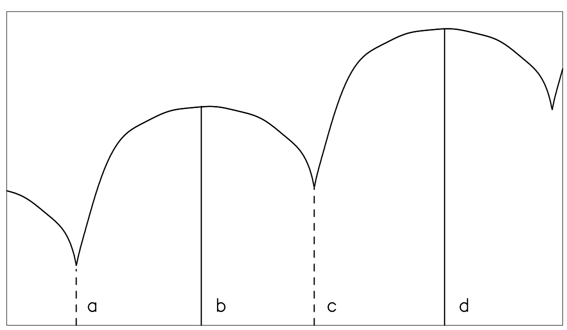

FIG. 3: The change in amplitude of successive expansion *maxima* is linked, via (2.15) & (2.16), to the hysteresis loop defined by $\oint pdV := \int_b^d pdV$. By contrast, the change in amplitude of successive expansion *minima* is related, via (2.20) & (2.21), to the hysteresis loop defined by $\oint pdV := \int_a^c pdV$.

It is interesting that a companion relationship to (2.15) can be derived for the change in the *minimum value* of the expansion factor at each successive bounce. Setting $M = \rho a^3$ as earlier, and noting from (2.2) that $\rho = \rho_c$ at the bounce,[5] we find $\delta M_{\rm bounce} = \delta(\rho_c a_{\rm min})^3$.

[5] We assume that the role of the curvature term can be neglected at the bounce which is a reasonable assumption provided matter satisfies the strong energy condition, $\rho + 3p \geq 0$, near the bounce.

The *work done* during an expansion-contraction cycle is

$$\delta W = \oint pdV = \int_{\text{expansion}} p\ dV + \int_{\text{contraction}} p\ dV\ . \tag{2.19}$$

Setting $\delta W + \delta M_{\text{bounce}} = 0$ we get

$$\delta(a_{\text{min}})^3 \equiv \left\{a_{min}^{(i)}\right\}^3 - \left\{a_{min}^{(i-1)}\right\}^3 = -\frac{1}{\rho_c}\oint pdV\ , \tag{2.20}$$

where the hysteresis loop is evaluated over one compete *expansion-contraction* cycle, namely

$$\oint pdV := \int_{a_{min}^{(i-1)}}^{a_{min}^{(i)}} pdV\ , \tag{2.21}$$

$a_{min}^{(i)}$ being the minimum value of the expansion factor in the i^{th} cycle; see figure 3.

Comparing (2.20) and (2.15) allows us to draw the following important conclusions:

- From (2.20) we find that the change in the *minimum* value of the expansion factor depends upon the value of the hysteresis loop, $\oint pdV$, and the cosmological matter density at the bounce, ρ_c, but is *insensitive to the nature of turnaround.* It is important to note that an *increase* in the minimum value of the expansion factor at the bounce, determined by (2.20), would lead to a corresponding *decrease* in the curvature parameter k/a_{min}^2 at the bounce, making it easier for a spatially closed universe to inflate even though the curvature term may have prevented inflation from occuring during earlier cycles (see also [3, 37]).

- By contrast, the change in the *maximum* value of the expansion factor, determined by (2.15), depends upon the nature of turnaround as well as $\oint pdV$, but is *insensitive to the density at the bounce.* In passing one might note that the value of n in (2.11) is not restricted to being an integer. Indeed if turnaround is sourced by a dynamical dark energy model such as the scalar field with potential $V(\psi) \propto \cos(\lambda\psi)$ then, as ψ rolls towards the negative minimum of V, kinetic terms will ensure that n never quite reaches $n = 0$, the value suggestive of a negative cosmological constant. Consequently one might expect a scalar field induced turnaround to mimic V_0/a^n with $V_0 < 0$ and $-2 \leq n < 0$.

- Note that in the conventional general relativistic framework described by (2.4) both the bounce as well as turnaround can be sourced by the (positive) curvature term. In

this case the condition for the bounce demands that, close to it, matter *violates* the strong energy condition $\rho + 3p \geq 0$. On the other hand the universe turns around if matter during much later times *satisfies* the strong energy condition ! It is easy to show that in this case (2.15) is replaced by $\delta a_{\rm max} = -\kappa \oint pdV$, with $\oint pdV$ evaluated as in (2.16), while (2.20) is replaced by a similar expression $\delta a_{\rm min} = -\kappa \oint pdV$, but where $\oint pdV$ is evaluated as in (2.21). Within such a conventional setting bouncing models with a scalar field were studied in [32]. However it seems unlikely that hysteresis could arise in such a scenario since $p \simeq -\rho$ was found to arise during contraction (inducing the bounce) and a similar equation of state arose again during the early stages of expansion. So the asymmetry between $p_{\rm contraction}$ and $p_{\rm expansion}$, which is an essential requirement for hysteresis, is virtually absent in this case.

- Note also that in a realistic cosmological model only some of the matter degrees of freedom driving cosmic expansion in (2.2) & (2.4) may display hysteresis. In this case one might expect (2.15) and (2.20) to generalize to

$$\delta \left(a_{\rm max}\right)^{3-n} = -\frac{\kappa}{A} \oint p_\alpha dV \ , \quad \delta \left(a_{\rm min}\right)^{3} = -\frac{1}{\rho_c} \oint p_\alpha dV \ , \tag{2.22}$$

where p_α denotes the pressure associated with the matter component displaying hysteresis *ie* $p_{\alpha,{\rm expansion}} \neq p_{\alpha,{\rm contraction}}$.

3. THE *BEATING* UNIVERSE

While the equations describing cosmological hysteresis appear to be model independent, the concrete value of the hysteresis loop $\oint pdV$ is related to the dynamics of the scalar field and, in particular, to the form of its potential $V(\phi)$. Let us consider some simple potentials which generate hysteresis.

[**1**] The 'chaotic' potential $V(\phi) = V_0\phi^{2k}$ gives rise to inflation for $k \leq$ few [33]. In addition, this potential can also serve to describe 'fuzzy' cold dark matter if $k = 1$ and V_0 is sufficiently small [38]. It is interesting that, depending upon the value of V_0, this potential can display a steady increase in the amplitude of successive cycles, as well as other interesting features such as *beats* and stochasticity. Figure 4 illustrates how the expansion factor grows with each successive cycle when turnaround is sourced by a positive curvature term (right panel) and a negative cosmological constant (left panel). The issue of *beats* and

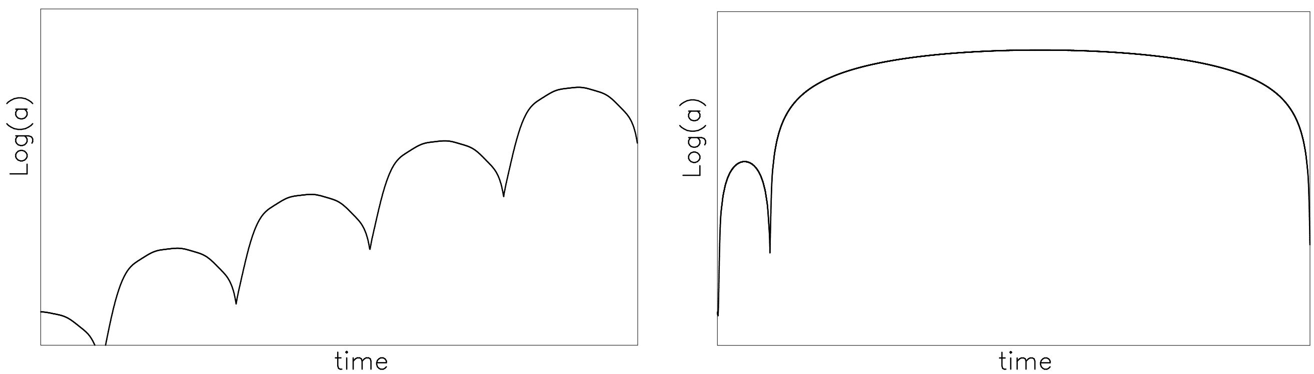

FIG. 4: The presence of hysteresis can *increase* the amplitude of successive expansion maxima in a cyclic universe sourced by the potential $V(\phi) = \frac{1}{2}m^2\phi^2$. In the left panel, cosmic turnaround is caused by the presence of a *negative* cosmological constant, so that $n = 0$ in (2.11). In the right panel, turnaround is caused by a (positive) curvature term, so that $n = 2$ in (2.11). In both cases the increased amplitude of successive cycles is described by the hysteresis equations (2.15) & (2.20). The successively increasing expansion cycles appear to endow the universe with an *arrow of time* even though the equations governing cosmological evolution are formally time reversable and there is no entropy production.

stochasticity will be discussed slightly later when we turn our attention to the $\cosh(\lambda\phi)$ potential.

A closed oscillating universe holds interesting consequences for the density parameter

$$\Omega - 1 = (aH)^{-2}\,. \tag{3.1}$$

As we just saw, cosmological hysteresis can lead to successively increasing expansion cycles which would draw the value of Ω towards unity. Indeed, (3.1) clearly demonstrates that, for an identical value of H, larger values of $a(t)$ will result in smaller values for $\Omega - 1$. Clearly, a universe with *strong* hysteresis ($\oint pdV$ is large and negative) will require comparatively fewer oscillatory cycles to reduce the value of $\Omega - 1$, as compared to one in which hysteresis was weaker. Thus in a universe with progressiviely increasing expansion cycles, such as the one shown in fig. 4, the value of the flatness parameter will gradually be drawn closer to unity gently ameliorating the flatness problem; see also [3].

For polynomial potentials $V \propto \phi^{2k}$ the end of inflation is marked by coherent oscillations of the scalar field and the resulting equation of state is

$$\langle w \rangle = \frac{k-1}{k+1} \ . \tag{3.2}$$

Consequently the density $\langle \rho_\phi \rangle \propto a^{-3(1+\langle w \rangle)}$ falls off *faster* than dust ($\rho \propto a^{-3}$) for $k > 1$, for instance $\langle \rho_\phi \rangle \propto a^{-4}$ in the case of the $\lambda\phi^4$ potential. As a result $M = \rho_\phi a^3$ is no longer a conserved quantity for $k > 1$! Nor for that matter is $M = \rho_\phi a^3$ conserved during inflation (when $\rho_\phi \simeq$ *constant*) or cosmological contraction (when $\rho_\phi \propto a^{-6}$). The formulae relating the quantum of hysteresis to the change in expansion maxima/minima however remain valid since they are derived on the basis of the conservation equations, $T^k_{i\,;k} = 0$, which are robust to changes in the form of the inflationary potential and mechanisms for making the universe turnaround and bounce. It therefore appears that cosmological hysteresis extends considerably beyond the domain of spatially closed FRW models for which it was originally explored [3].

[2] Another interesting example of cosmological hysteresis and cyclicity is provided by the potential ($\tilde{m}_P^{-1} = \sqrt{8\pi G}$)

$$V = V_0(\cosh \lambda\phi/\tilde{m}_P - 1) \ , \tag{3.3}$$

whose suitability for being a dark matter candidate was discussed in [39, 40].
For $\lambda\phi \ll 1$, $V \propto \lambda^2\phi^2$ and the field oscillates as pressureless matter with $\langle p \rangle = 0$.
For $\lambda\phi \gg 1$, $V \simeq \frac{1}{2}V_0 \exp\left(\lambda\phi/\tilde{m}_P\right)$ and the behaviour of the field can be assessed using the slow roll parameters

$$\epsilon = \frac{\tilde{m}_P^2}{2}\left(\frac{V'}{V}\right)^2 \simeq \frac{\lambda^2}{2} \ , \quad \eta = \tilde{m}_P^2\left(\frac{V''}{V}\right) \simeq \lambda^2 \ . \tag{3.4}$$

Two extreme cases deserve special mention.

- If $\lambda \ll 1$ then $\{\epsilon, \eta\} \ll 1$ and the hysteresis loop $\oint pdV$ has a large absolute value signalling *strong* hysteresis and a steady growth in amplitude of successive cycles.

- For $\lambda \geq 1$, on the other hand, the slow-roll parameters are large and inflation with its associated regimes (2.9) & (2.10) need not occur. Surprisingly, $|\oint pdV| \neq 0$ even in this case, and the universe can display striking behaviour for moderate values of the control parameter $1 \lesssim \lambda \lesssim$ *few*.

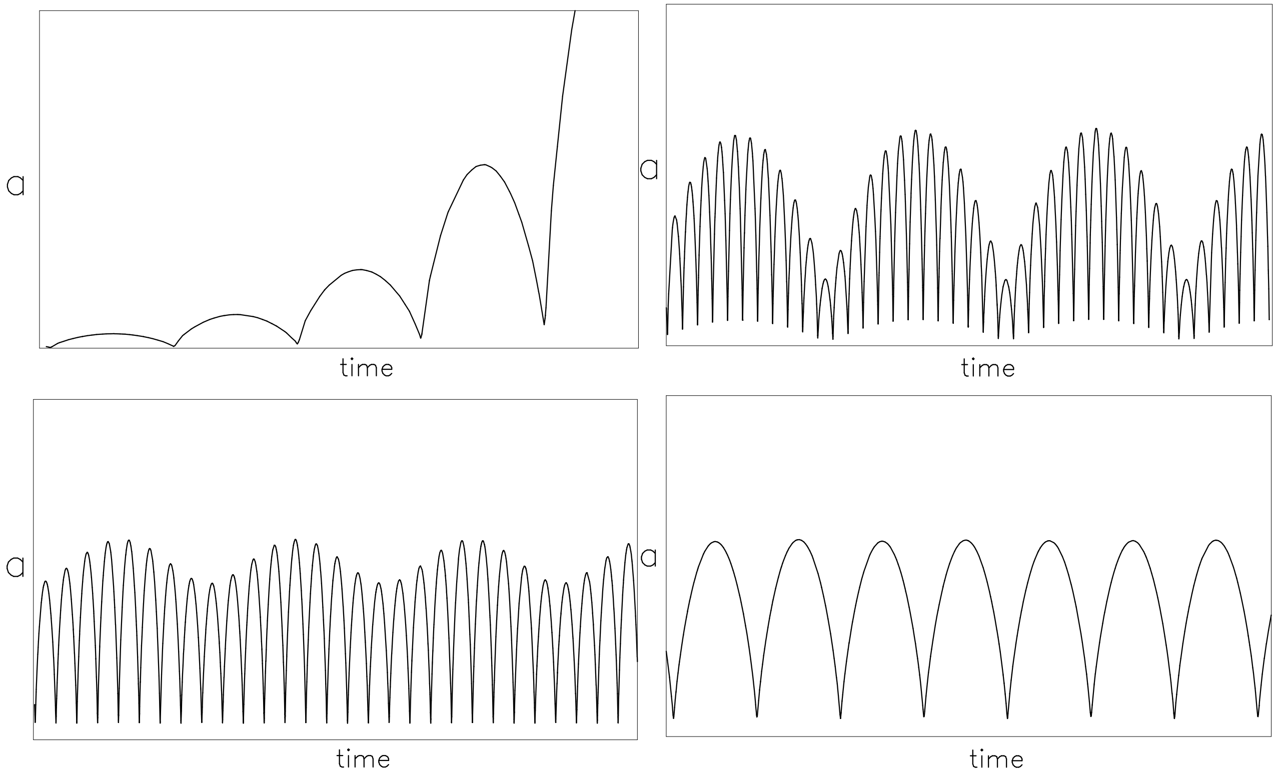

FIG. 5: A cyclic universe sourced by the potential $V = V_0(\cosh \lambda\phi/\tilde{m}_P - 1)$. Increasing the value of the control parameter λ causes cyclic behaviour to change dramatically, as illustrated in the four panels. In all cases turnaround is caused by a negative cosmological constant: $n = 0$ in (2.11), while the control (steepness) parameter has values $\lambda = 2,\ 2.5$ (upper left and right) and $\lambda = 4,\ 6$ (lower left and right). As the control parameter increases the behaviour of the universe undergoes a remarkable change. For $\lambda \lesssim 2$ the amplitude of successive cycles increases (top left), with the increase being larger for smaller values of λ. For moderate values of λ (top right and bottom left panels) the universe displays a quasi-periodic pattern reminiscent of *beats* in sound waves. During *beats* the value of $\oint pdV$ is negative during the first half of the larger (parent) cycle and positive during the second half. Beats gradually disappear as the value of λ is increased. The universe now begins to show oscillatory behaviour in which all cycles have roughly the same amplitude and duration signifying $\oint pdV \simeq 0$ (bottom right). In all panels the amplitude of successive cycles is governed by the simple formulae (2.17) and (2.20).

The expansion factor for the cosh potential is illustrated in figures 5 & 6. Figure 5 corresponds to the case when turnaround is sourced by a negative cosmological constant while in fig. 6 turnaround is sourced by positive (spatial) curvature. Comparing the two figures we see similarities in the behaviour of $a(t)$ as well as interesting differences. In both cases, moderately small values of the steepness parameter λ give rise to a steady increase in amplitude of successive cycles. This arises because the hysteresis loop $\oint pdV$ is a negative quantity which leads to a progressively larger universe. Note that while a spell of inflation does ensure $\oint pdV < 0$, successively larger cycles do not necessarily imply the existence of inflation. Indeed for the scalar field models which we have studied, the presence of hysteresis appears to be a rather general phenomenon and all that is required for the occurance of successively larger expansion cycles is $\oint pdV < 0 \Rightarrow p_{\rm expansion} < p_{\rm contraction}$. The inflationary universe occupies the *extreme end* of this relationship since $w_{\rm expansion} \simeq -1$ and $w_{\rm contraction} \simeq 1$, as illustrated in figure 2.

As the value of λ is increased a fundamental difference can be discerned in the behaviour of $a(t)$ in figures 5 and 6. In figure 5 (second and third panels) one sees a modulation in the amplitude of successive cycles suggestive of *beats* in an acoustic system. *Beats* in the expansion of the universe are characterized by a two fold cyclic pattern with smaller duration (daughter) cycles nested within a large (parent) cycle. The origin of *beats* can be traced to periodic changes in the value of the hysteresis loop $\oint pdV$. During the first half of the parent cycle, $\oint pdV < 0$, which leads to a steady increase in the expansion maxima of successive (daughter) cycles. As the parent cycle reaches its maximum value the hysteresis loop changes sign so that $\oint pdV > 0$ during the next half cycle. This leads to a steadily diminishing amplitude of daughter cycles during the next (parent) half-cycle in accordance with (2.15). This behaviour is repeated in a self-similar manner during subsequent parent and daughter cycles. As in the case of acoustic beats, the daughter cycle with the smallest amplitude is located at the boundary of two parent cycles. But unlike the acoustic case, the modulation in the value of $a_{\rm min}$ for daughter cycles is more graded than that in $a_{\rm max}$, with the latter showing more pronounced changes in amplitude during a given parent cycle.

The *beating* universe is robust to small changes in the value of the steepness parameter. Large values ($\lambda \gtrsim$ few) however lead to diminished hysteresis and an end to the *beats* phenomenon. In this case expansion maxima and minima equalise and the oscillatory behaviour of the universe (fig. 5 lower right panel) begins to resemble that shown in figure 1

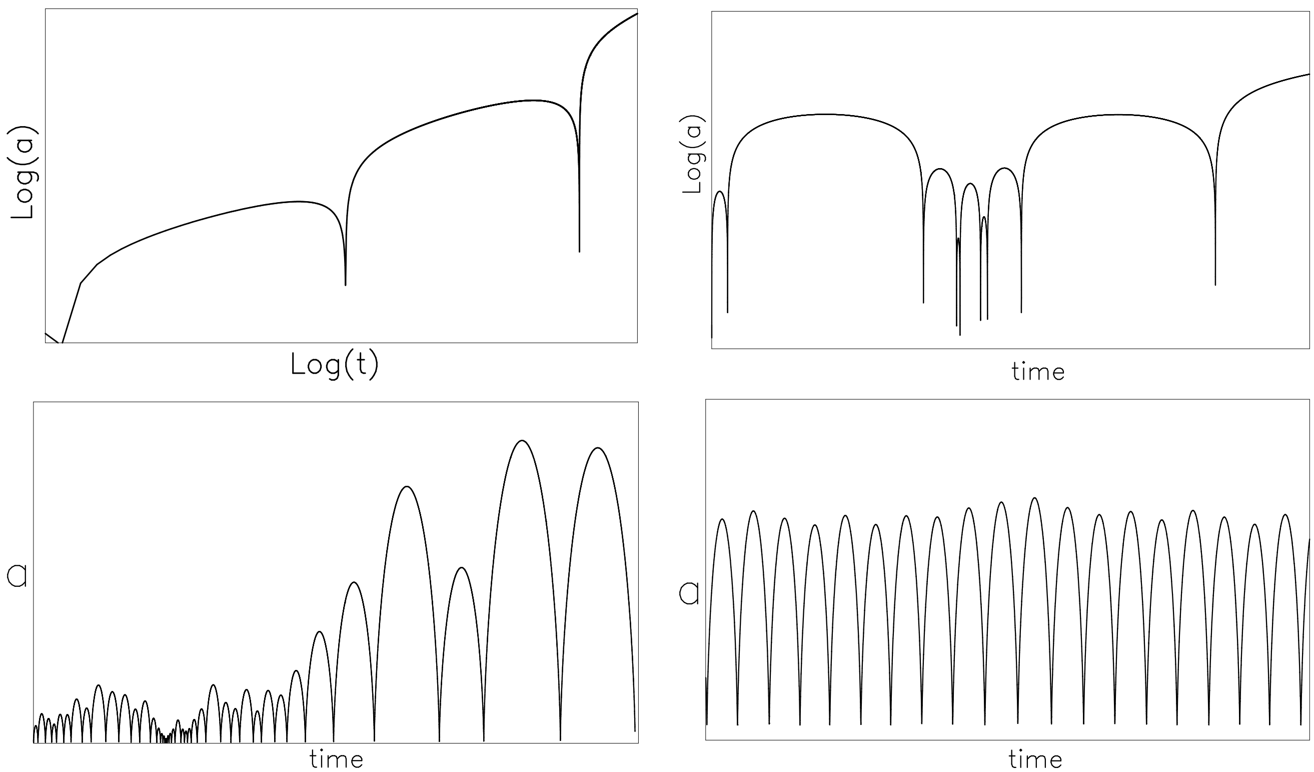

FIG. 6: A cyclic universe sourced by the potential $V = V_0(\cosh\lambda\phi/\tilde{m}_P - 1)$. Increasing the value of the control parameter λ causes cyclic behaviour to change dramatically, as illustrated in the four panels. In all cases turnaround is caused by a positive curvature term: $n = 2$ in (2.11), while the control (steepness) parameter has values $\lambda = 1.5$, 2 (upper left and right) and $\lambda = 3.3$, 6 (lower left and right). The top two panels show the expansion factor in logarithmic units. In the top left panel each cyclic epoch is larger in amplitude and duration than its predecessor, indicating $\oint pdV < 0$. As the steepness parameter is increased the behaviour of successive cycles becomes highly stochastic (top right and bottom left) which is indicative of the fact that the hysteresis loop can be negative ($\oint pdV < 0$) as well as positive ($\oint pdV > 0$) during cycles. Once more we find larger amplitude cycles to have a longer duration. Further increase of λ leads to a stage when hysteresis is virtually absent and all cyclic epochs become similar. In all panels the amplitude of successive cycles is governed by the simple formulae (2.18) and (2.20).

for a cosmological model in which expansion and contraction epochs are identical, so that $p_{\rm expansion} = p_{\rm contraction}$ and $\oint pdV = 0$.

The existence of the *beats* phenomenon appears to depend sensitively on the nature of

turnaround. Indeed, it appears that beats are entirely absent if turnaround is sourced by a (positive) spatial curvature term. It is interesting that for a closed universe the phenomenon of beats is effectively replaced by that of *stochasticity*, as can readily be seen by comparing figure 6 with figure 5. During stochasticity larger expansion maxima are interspersed with smaller ones. Such a situation arises if both $\oint pdV < 0$ as well as $\oint pdV > 0$ can occur during consecutive cycles, and is the physical basis underlying *stochasticity*.[6]

The origin of stochasticity lies in the fact that for moderately steep potentials $\lambda \sim O(1)$, the pressure of the scalar field during expansion can, on occasion, exceed that during contraction. This is especially true when a curvature term is present, since in that case $H^2 = \kappa\rho - k/a^2$, and the value of the Hubble parameter is smaller than it would be if k/a^2 were replaced by a negative cosmological constant. This situation leads to less friction (anti-friction) during expansion (contraction) in the scalar field equation of motion (2.8), altering the nature of hysteresis and accomodating the possibility $p_{\rm expansion} > p_{\rm contraction}$, and $\oint pdV > 0$. Equation (2.18) then leads to $\delta a_{\rm max} < 0$, in other words the amplitude of successive cycles can *decrease* as well as increase ! This is shown in figure 6 which also informs us that the duration of a cycle is related to its amplitude and that larger amplitude cycles are of longer duration. (In the universe of figure 5, on the other hand, all cycles are of roughly equal duration.) Further increase of the control parameter ($\lambda \gg 1$) causes the potential to steepen to such a degree that expansion and contraction epochs become roughly similar, so that $p_{\rm expansion} \simeq p_{\rm contraction}$. In this case hysteresis is virtually absent, $\oint pdV \simeq 0$, and the amplitude of successive cycles equalizes giving $\delta a_{\rm max} \simeq 0$ (figure 6 bottom right).

Note too that stochasticity is usually present only if the potential is too steep to sustain inflation. In the presence of inflation one finds $\oint pdV < 0$, with the result that the amplitude of successive cycles grows with time and stochasticity disappears. Note too that all of the above features, namely: (i) monotonically increasing expansion cycles (for small values of the control parameter), (ii) *Beats* and stochasticity (for moderate values of the control parameter) exist also for potentials other than $\cosh(\lambda\phi)$, including $V \propto \phi^{2k}$ commonly associated with inflation.

[6] The fact that stochasticity will be absent in the cosmological model shown in figure 5 can be understood from the following argument. The equations of motion for this model (in which recollapse is sourced by a negative cosmological constant) can be written in terms of $\phi, \dot\phi, H$ governed by a single constraint. Phase space is therefore two dimensional and stochasticity is absent due to topological reasons [41].

A complementary means of generating hysteresis and the associated increase in consecutive cycles, is by allowing the fluid filling the universe to be viscous and dissipative [1]. If ζ is the coefficient of bulk viscosity then the fluid pressure changes from its equilibrium value p_0 to $p = p_0 - 3\zeta H$. Consequently $p < p_0$ during expansion while $p > p_0$ during contraction. The resulting growth in entropy causes sucessive expansion cycles to be larger. The possibility that bulk viscosity might drive cosmic acceleration has been studied in [42], and a recent attempt linking entropy production to increased expansion cycles is discussed in [43].

A central difference between this approach and ours is that our system of equations (2.1), (2.8) & (2.11) is dissipationless and therefore formally time reversible. Yet the presence of cosmological hysteresis ($\oint pdV \neq 0$) endows the universe with a plethora of new features including *beats*, stochasticity as well the possibility of a regular increase in the amplitude of consecutive cycles. The reason for this proliferation of possibility rests in the following.

As pointed out in [3], the presence of hysteresis is closely linked to the ability of the field ϕ to oscillate. Indeed oscillations appear to be vital for the existence of hysteresis since they play the important role of *mixing* the field in phase-space $\{\dot{\phi}, \phi\}$ due to which the value of $\{\dot{\phi}, \phi\}$ when the universe turns around and contracts is almost uncorrelated with its phase space value when the field $\phi(t)$ began oscillating. This phase-space mixing ensures that (during contraction) the scalar field almost never rolls up $V(\phi)$ along the same phase-space trajectory down which it descended (during expansion), thereby ensuring $P_{\text{contraction}} \neq P_{\text{expansion}}$ and $\oint pdV \neq 0$.

We therefore conclude that $V(\phi)$ must have a well defined minimum value in some region of configuration space in order to allow the possibility of hysteresis. Consider as an alternative the potential $V(\phi) \propto \phi^{-\alpha}$ which does not possess a minimum and is a popular candidate for dark energy [44]. In this case no oscillatory phase is present which will reverse the sign of $\dot{\phi}$ during contraction, causing $\phi(t)$ to roll up its potential instead of down. As a result $V(\phi)$ continuously decreases in value when the universe expands as well as contracts. The role of the potential during successive cycles therefore soon becomes negligible, and cosmological dynamics becomes solely governed by the kinetic energy $\dot{\phi}^2 \propto a^{-6}$. The universe, in this case, behaves as if it were dominated by a perfect fluid with the stiff equation of state $P = \rho$ and there is no possibility of hysteresis, as demonstrated in figure 7.

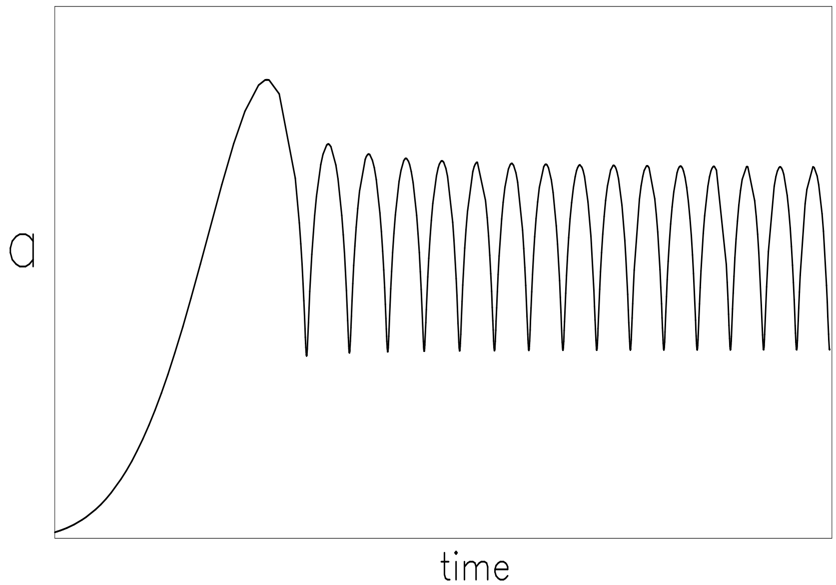

FIG. 7: The expansion factor of a spatially closed universe dominated by a scalar field with the potential $V \propto \phi^{-\alpha}$, $\alpha > 0$. The universe soon settles into an oscillatory mode in which all cycles are of equal amplitude indicating $\oint p dV = 0$.

4. SCALAR FIELD DYNAMICS DURING CONTRACTION

An important question which arises in connection with the bouncing scenario is whether the scalar field can rise high enough on its potential (during contraction) to provide an adequate number of inflationary e-folds during the ensuing round of (post-bounce) expansion. This value can be easily estimated as follows.

As discussed earlier, the inflationary scalar field ($V = m^2\phi^2/2$) in a cyclic universe passes through three successive regimes during which: (i) $\phi \gtrsim m_P \equiv G^{-1/2}$ and the field slow-rolls down its potential resulting in $p \simeq -\rho$, (ii) $\phi \lesssim m_P$ and the motion of the field becomes oscillatory leading to $\langle p \rangle \simeq 0$. The oscillatory regime exists during expansion as well as the early stages of contraction. (iii) During the late stages of contraction the scalar field amplitude grows to $\phi \sim m_P$. The field now stops oscillating and begins to climb up its potential leading to the *stiff* equation of state $p \simeq \rho$. (This is in response to anti-friction in (2.8) since $H < 0$ during contraction.) Below we shall provide a simple analytical estimate of how far up its potential the field can climb before beginning its descent, soon after the universe has bounced, and the universe starts to expand once more.

It is useful to note in this connection, the following exact expression describing the motion

of a massive scalar field in a spatially flat universe which expands/contracts as a power law $a(t) \propto t^p$, where $p = 2/3(1+w)$ and w is the equation of state:

$$\frac{\phi(t)}{m_P} = \sqrt{\frac{t}{a^3(t)}}\left\{AJ_\nu(mt) + BY_\nu(mt)\right\} , \quad \nu = \frac{1-w}{2(1+w)} . \tag{4.1}$$

During the oscillatory dust-like phase, $\nu = 1/2$, and

$$\phi(t) \propto \frac{\cos(mt+\theta)}{a(t)^{3/2}} \tag{4.2}$$

where θ is a phase constant. Equation (4.2) informs us that the amplitude of the scalar field, averaged over a certain number of oscillations, increases/decreases in a contracting/expanding universe as $\langle\phi^2\rangle \propto a^{-3}$. During contraction, once the field value reaches $\phi \sim m_P$, the oscillatory regime ceases and the scalar field begins to climb up its potential. This marks the commencement of the *kinetic regime* during which the equations of motion begin to be dominated by the kinetic energy of ϕ, resulting in $w \simeq 1$, $\nu = 0$ in (4.1), and the solution

$$\frac{\phi(t)}{m_P} = AJ_0(mt) + BY_0(mt) , \quad \text{where } J_0 \simeq 1,\ Y_0 \simeq \frac{2}{\pi}\ln(mt) , \quad \text{when } mt \ll 1 . \tag{4.3}$$

In other words $\phi = \dot{\phi}_{in} t_{in} \ln t/t_{in}$, where $\dot{\phi}_{in}$ and t_{in} are initial values of the scalar field velocity and the cosmic time at the commencement of the kinetic regime[7].

Assuming that the bounce occures at the Planck energy and remembering that $\phi \sim m_P$ when oscillations end, we get the following result for the scalar field amplitude at the instant of the bounce

$$\phi_b = \frac{m_P}{\sqrt{12\pi}} \ln\frac{m_P}{m}. \tag{4.4}$$

(The actual value is smaller if the bounce occures at energies below the Planck scale.) It is worth noting that (4.4) is valid for any potential which is less steep than the exponential[8] and in this case the ratio H_B/H_{in} of Hubble parameters at the bounce and at the commencement of the kinetic regime, replaces m_P/m in the logarithm of (4.4) so that

$$\phi_b = \frac{m_P}{\sqrt{12\pi}} \ln\frac{H_B}{H_{\rm in}}. \tag{4.5}$$

---

[7] This equation can also be derived by noting that $\ddot{\phi} + 3H\dot{\phi} \simeq 0$ during the kinetic regime.
[8] In the derivation we used the massless approximation which fails for steep potentials [45].

While the above expression is quite general and holds for a fairly wide class of inflationary potentials, the number of e-folds during the post-bounce inflationary stage does depend upon the concrete form of the inflaton potential. Consider for instance the chaotic inflationary potential, $V = m^2\phi^2/2$ with $m = 10^{-6}m_P$, which is in excellent agreement with observations. Substituting $m = 10^{-6}m_P$ into (4.4) one obtains $\phi_b \sim 2m_P$, and an embarassingly small value for the number of inflationary e-folds $N = 2\pi\phi_b^2 \sim 25$. Surprisingly this does not indicate that this model can be ruled out, since, to the scalar field value at the bounce, ϕ_b, one needs to add ϕ_{in} – the 'initial' value of the scalar field at the commencement of the kinetic regime. The value of ϕ_{in} depends upon the phase of the oscillatory scalar field and ranges from $-m_P$ to m_P. Moreover, immediately after the bounce the absolute value of the scalar field continues to increase despite the fact it now encounters a large amount of friction (the post-bounce value of H being positive). Indeed, our numerical simulations[9] indicate that while the value of the scalar field at the bounce is only $2.5\, m_P$, the field manages to climb to the significantly higher value $4.6\, m_P$ soon after the bounce, resulting in the observationally comfortable value $N \sim 130$.

5. THE BEHAVIOUR OF ANISOTROPY IN AN OSCILLATING UNIVERSE

A central issue in cosmology concerns the class of initial conditions which can give rise to a universe resembling our own. This issue was highlighted in [46] which showed that isotropic models were a set of zero measure in the space of all homogenous solutions of the Einstein equations [47]. Subsequently several mechanisms were identified, including cosmological particle creation [48], which might successfully dissipate a large initial anisotropy within short span of time, so that a universe which started out being highly anisotropic would rapidly isotropize to a FRW space-time. Perhaps the most successful of these mechanisms is inflation, which possesses a no hair property which allows the universe to inflate from a fairly general class of initial conditions [49, 50]. In the context of the present paper we would like to ask the question as to how a large amount of anisotropy might impact the behaviour of a cyclic universe. For this purpose we shall focus on a Bianchi I universe which expands

[9] Although our analysis is in broad agreement with that of [10] our results for N are somewhat smaller than those in that paper. We believe this is due to a small typo in [10] due to which the value of the scalar field at the bounce is overestimated.

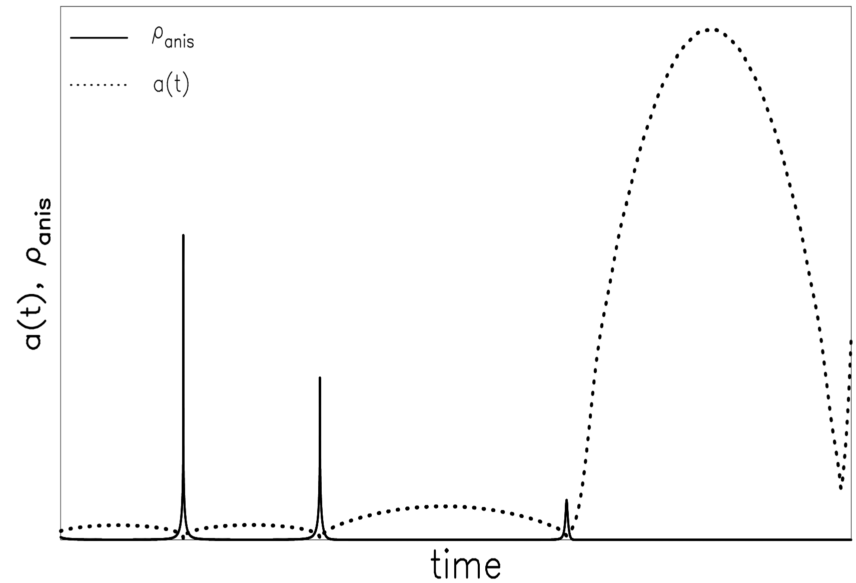

FIG. 8: The expansion factor (dotted) of a spatially anisotropic Bianchi I universe dominated by a massive scalar field. The anisotropy density (solid), given by $\rho_{\rm anis} = \Sigma/a^6$, first increases and then decreases at the time of each bounce. The hysteresis loop for this universe is negative $\oint pdV < 0$, with the result that the amplitude of successive cycles gradually increases. This leads to the waning of anisotropy with each successive cycle with the result that $\rho_{\rm anis}$ becomes vanishingly small at the commencement of the fifth cycle and cannot be resolved on the scale of the figure.

at different rates along the three spatial directions and for which the line element is

$$ds^2 = dt^2 - R_1^2(t)dx^2 - R_2^2(t)dy^2 - R_3^2(t)dz^2 \ . \tag{5.1}$$

Introducing the directional expansion rate

$$H_i = \dot{R}_i/R_i, \quad i = 1, 2, 3 \tag{5.2}$$

and the mean expansion factor $a(t) = (R_1 R_2 R_3)^{1/3}$ we find the mean expansion rate

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\sum_{i=1}^{3} H_i \ , \tag{5.3}$$

in terms of which the expression for the anisotropy is simply

$$\sum_{\alpha=1}^{3}(H_\alpha - H)^2 = \frac{\Sigma}{a^6} \ . \tag{5.4}$$

The resulting (0-0) Einstein equation

$$3H^2 = 8\pi G\rho + \Lambda + \frac{\Sigma}{a^6} \tag{5.5}$$

contains the anisotropy in the RHS as if it were an effective energy density with the *stiff* equation of state $P_{\rm anis} = \rho_{\rm anis} = \Sigma/a^6$ [50]. In order to assess the behaviour of anisotropy in a cyclic scenario we shall assume $\Lambda < 0$, which permits the universe to turnaround and contract. As before, we also assume that in the vicinity of the 'Big Bang' extra-dimensional effects [4] modify the FRW equations to

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\rho\left\{1 - \frac{\rho}{\rho_c}\right\} + \frac{\Lambda}{3} + \frac{\Sigma}{a^6}\ , \\ \frac{\ddot a}{a} &= -\frac{4\pi G}{3}\left\{(\rho+3p) - \frac{2\rho}{\rho_c}(2\rho+3p)\right\} + \frac{\Lambda}{3} - 2\frac{\Sigma}{a^6}\ . \end{aligned} \tag{5.6}$$

Our results shown in figure 8 indicate that, if the scalar field is not too heavy ($m \lesssim m_{\rm Pl}$), then the oscillating universe displays monotonically increasing cycles which cause the anisotropy density $\rho_{\rm anis} = \Sigma/a^6$ to decrease from cycle to cycle. The necessary condition for the gradual disappearence of anisotropy and the isotropisation of the universe is therefore $\oint pdV < 0$, since this guarantees that successive cycles are larger in amplitude to previous ones. Note that this does not require the presence of inflation, although a short burst of inflation at the start of every cycle would clearly assist the abatement of anisotropy.

6. DISCUSSION

In the present paper we have shown how a cyclic universe containing a self-interacting minimally coupled scalar field exhibits interesting relationships, (2.15) & (2.20), linking the change in the expansion factor at turnaround/bounce to the net work-done during a given expansion-contraction cycle: $\oint pdV$. This relationship is quite general and requires only that the universe turnaround and bounce, so that cyclicity is maintained. It is legitimate to ask whether such a scenario can be realised within the framework of a universe which accelerates at late times, and therefore resembles our own. The demand that cyclicity makes of cosmic acceleration is that the latter be transient since otherwise our current accelerating phase would become a permanent feature of our universe, preventing turnaround. It is well known that a transiently accelerating phase can occur in several distinct models of dark energy [28–31]. However, an investigation of the presence of hysteresis in such models lies somewhat beyond the scope of the present paper and we do not discuss it here.

Another important issue, left untouched in the present paper, concerns the development of density inhomogeneities. The universe today is characterized by structure which is sig-

nificantly nonlinear on scales shorter than a few Mpc. The criterion $\delta\rho/\rho \sim 1$ marks the onset of nonlinearity and the comoving nonlinear length scale $k_{\rm NL}^{-1}$ can be defined from the equality [51]

$$\langle(\delta\rho/\rho)^2\rangle^{1/2} = D_+(t)\left(4\pi\int_0^{k_{\rm NL}} P(k)k^2dk\right)^{1/2} = 1 \tag{6.1}$$

where $D_+(t)$ is the growing mode of the linearized density contrast and $P(k)$ is the power spectrum of linear density fluctuations. For power law spectra $P(k) \propto k^n$

$$k_{\rm NL}^{-1} \propto D_+^{2/(n+3)}(t) \ , \tag{6.2}$$

and, since $D_+(t)$ grows with time, it follows that the nonlinear length scale grows continuously from small initial values at early times to $k_{\rm NL}^{-1} \sim 10$ Mpc currently. The growth of the nonlinear length scale with time reflects the fact that gravitational clustering takes place hierarchically in gravitational instability scenario's based on cold/warm dark matter. The power spectrum in such scenario's is characerized by the local spectral index $n_{\rm eff} = d\log P(k)/d\log k$ whose value, in the case of cold dark matter, ranges from $n_{\rm eff} \simeq 1$ on scales $\gg 10$ Mpc to $n_{\rm eff} \simeq -3$ on scales $\ll 1$ Mpc. From (6.2) we find that the growth in $k_{\rm NL}^{-1}$ will be affected both by the change in value of the slope $n_{\rm eff}(t)$ (as successively larges scales become nonlinear) and the behaviour of $D_+(t)$. The latter is governed, in the absence of pressure and on scales significantly smaller than the Hubble length, by the Jeans-type equation

$$\ddot{D}_+ + 2H\dot{D}_+ - 4\pi G\bar{\rho}D_+ = 0 \ , \tag{6.3}$$

from which we learn that the Hubble parameter behaves like a damping term during expansion, when $H > 0$, causing linearized perturbations to grow much slower than they would in a static universe[10]: $D_+ \propto \exp\sqrt{4\pi G\bar{\rho}}t$. During contraction, $H < 0$, the situation is reversed since the (negative) Hubble parameter now plays the role of an anti-damping term which significantly speeds up gravitational instability relative to expansion. This means that the nonlinear length scale will grow rapidly during contraction eventually overtaking the comoving Hubble length $(aH)^{-1}$ which *decreases* with time in a contracting universe. One therefore suspects that a universe resembling ours will become strongly inhomogeneous

[10] In a realistic scenario which incorporates both DE and turnaround, such as [28], density perturbations would be expected to grow as $D_+ \propto a(t)$ during matter domination, slowing considerably when the universe began to accelerate, and speeding up once more when the universe turned around and contracted.

during contraction and it is not clear whether the assumptions which went into the derivation of the bouncing equations (2.1) will hold in this case. (Since most bouncing solutions have been derived within the FRW setting and rely on assumptions of homogeneity [19], these concerns are also relevant to them.)

The above argument necessarily applies to an *old* universe resembling ours. It need not be true for a cyclic epoch of much shorter duration [11]. If a shorter cyclic epoch preceeded ours then gravitational instability would have had insufficient time to grow to nonlinear values and the universe during contraction might persist in being quasi-homogeneous (see [52] for an analysis of linealized perturbations in cyclic models). Such a situation is illustrated by the right panel of figure 4, the first three panels of figure 6 and by figure 4 of [3].

To summarize, we have demonstrated the presence of hysteresis in spatially flat and closed FRW cosmologies filled with a self-interacting scalar field. For these models we have established a rather simple relationship between the growth in the expansion factor and the quantum of hysteresis, namely (2.15). We have shown that, depending upon the value of the hysteresis loop, the universe can exhibit a wide format of behaviour including progressively larger expansion cycles (when $\oint pdV < 0$) as well as stochasticity. An early inflationary epoch is not an essential prerequisite for the existence of increasing expansion cycles. What is required in this case is that the hysteresis loop be negative, $\oint pdV < 0$, and even a small asymetry between the fluid pressure during expansion and contraction can accomplish this. In such situations, when $w_{\rm expansion} < w_{\rm contraction}$ but $w_{\rm expansion} > -1$ we sat thay hysteresis is small, in contrast to the situation portrayed in fig 2 when $w_{\rm expansion} \simeq -1$ and hysteresis is large. As demonstrated in this paper, even in situations with small hysteresis, the moderate increase in successive cycles can draw the cosmological density parameter towards unity, gently ameliorating the flatness problem. The same is true for spatial anisotropy whose value declines in a universe with growing cycles. The presence of Cosmological hysteresis can also adorn the universe with quasi-regular oscillations, or *beats*, reminiscent to those in acoustic systems. One should note here that the phenomenon of beats (and stochasticity)

[11] The above argument can also be circumvented if, during the bounce, the universe gets 'recycled' so that it embarks on its next expansion cycle with a new set of coupling constants and a different inflationary potential $V(\phi)$. A related possibility exists within the landscape paradigm if the inflaton potential is multi-dimensional since the inflaton field ϕ can sample different pieces of the potential during different cycles, as pointed out in [11].

is not peculiar to the $\cosh(\lambda\phi)$ potential but has been observed in other potentials as well, including the potential $V = \frac{1}{2}m^2\phi^2$ associated with chaotic inflation.

Our treatment has focussed on scalar fields possessing canonical kinetic terms and coupling minimally to gravity. We find it intriguing that while the system which we study is fully relativistic, its broad dynamical features can be enapsulated by the well known non-relativistic thermodynamic expression $\delta E = \oint pdV$. It would therefore be interesting to explore whether the phenomenon of hysteresis is more general and extends to cosmologies in which some of the assumptions of this paper are relaxed, such as the non-canonical scalar field models associated with DBI inflation [53], k-essence [54], ghost condensate models [55], cosmologies with interacting fluid components [56], etc. One could also ask whether cosmological hysteresis exists for scalar fields which couple non-minimally to gravity, for instance through $\xi R\phi^2$, Brans-Dicke and field derivative type couplings [57], or in cosmological models featuring non-local gravity [26]. We also leave untouched the interesting issue of quantum stability of a cyclic model displaying hysteresis, which touches on issues beyond the scope of the present paper [58]. Finally, it may also be worth enlarging the present analysis to (higher-dimensional) anisotropic models in which some of the spatial directions expand while others contract, and ask whether a form of hysteresis might exist in this case too.

Acknowledgments

We acknowledge useful discussions with Tarun Saini, Yuri Shtanov, Parampreet Singh and Sanil Unnikrishnan.

---

[1] R. C. Tolman, *Relativity, Thermodynamics and Cosmology*, Clarendon Press, Oxford (1934).

[2] A. Borde, A.H. Guth and A. Vilenkin, Phys. Rev. Lett. **90**, 151301, 2003.

[3] N. Kanekar, V. Sahni and Yu. Shtanov, Phys. Rev. D **63** 083520 2001 [astro-ph/0101448].

[4] Yu. Shtanov and V. Sahni, Phys. Lett. B **557** 1 2003 [e-Print: gr-qc/0208047].

[5] A. Ashtekar, T. Pawlowski and P. Singh, Phys. Rev. D **73**, 124038, 2006 [e-Print: gr-qc/0604013].

[6] G. Calcagni, Phys. Rev. D **71**, 023511, 2005 [e-Print: gr-qc/0410027]; E. J. Copeland, J. E. Lidsey and S. Mizuno, Phys. Rev. D **73**, 043503, 2006 [e-Print: gr-qc/0510022]; P. Singh,

Phys. Rev. D **73**, 063508, 2006 [e-Print: gr-qc/0603043]. T. Stachowiak, and M. Szydlowski, Phys. Lett. B **646**, 209, 2007 [e-Print: gr-qc/0610121]; H. Maeda, V. Sahni, Yu. Shtanov, Phys. Rev. D **76**, 104028, 2007, Erratum-ibid.D80:089902,2009. [e-Print: arXiv:0708.3237]

[7] A. Ashtekar, T. Pawlowski, P. Singh, K. Vandersloot, Phys. Rev. D **75**, 024035,2007 [e-Print: arXiv:gr-qc/0612104]; P. Singh, F. Vidotto, Phys. Rev. D **83**, 064027, 2011 [e-Print: arXiv:1012.1307].

[8] M. J. Rees, The Observatory **89**, 193, 1969.

[9] A. A. Starobinsky, JETP Lett. **30**, 719, 1979

[10] Y-S. Piao and Y-Z. Zhang, Nucl.Phys.B **725**, 265, 2005 [e-Print: gr-qc/0407027];

[11] Y-S. Piao, Phys. Rev. D **70**, 101302, 2004 [e-Print: hep-th/0407258]; Y-S. Piao, Phys. Lett. B **677**, 1, 2009; J. Zhang, [e-Print:arXiv:1102.5395];

[12] P.W. Graham, *et al.*[e-Print:arXiv:1109.0282].

[13] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **64**, 123522 2001 [arXiv:hep-th/0103239]; J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt and N. Turok, Phys. Rev. D **65**, 086007 2002 [arXiv:hep-th/0108187]. P. J. Steinhardt and N. Turok, Phys. Rev. D **65**, 126003 2002 [arXiv:hep-th/0111098]; Science **296**, 1436 2002. J. Khoury, P. J. Steinhardt and N. Turok, Phys. Rev. Lett. **91**, 161301 2003 [arXiv:astro-ph/0302012]. J. Khoury, P. J. Steinhardt and N. Turok, Phys. Rev. Lett. **92**, 031302 2004 [arXiv:hep-th/0307132].

[14] E. I. Buchbinder, J. Khoury and B. A. Ovrut, Phys. Rev. D **76**, 123503 (2007) [hep-th/0702154].

[15] R. Kallosh, L. Kofman and A. Linde, Phys. Rev. D **64**, 123523, 2001 [arXiv:hep-th/0104073]; J. Khoury, B. A. Ovrut, P. J. Steinhardt, N. Turok, arXiv:hep-th/0105212; R. Kallosh, J. U. Kang, A. Linde, V. Mukhanov, JCAP 0804:018,2008 [arXiv:0712.2040]; A. Linde, V. Mukhanov, A. Vikman, JCAP 1002:006,2010 [arXiv:0912.0944]; B. Xue and P. J. Steinhardt, Phys. Rev. D **84**, 083520 (2011) [arXiv:1106.1416].

[16] J. -L. Lehners, Class. Quant. Grav. **28**, 204004 (2011) [arXiv:1106.0172].

[17] J. D. Barrow and M. P. Dąbrowski, MNRAS **275**, 850 (1995). R. Dürrer and J. Laukenmann, Class. Quant. Grav. **13**, 1069 (1996).

[18] T. Biswas, e-Print: arXiv:0801.1315 T. Biswas and S. Alexander, Phys. Rev. D **80**, 043511, 2009 [e-Print: arXiv:0812.3182]

[19] M. Novello and S.E.Perez Bergliaffa, Phys.Rept. **463** 127, 2008. [e-Print: arXiv:0802.1634]

[20] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Spacetime*, Cambridge University Press, Cambridge (1973).

[21] C. Cattoen and M. Visser, Class. Quant. Grav.**22**, 4913, 2005 [e-Print: gr-qc/0508045]

[22] P. Kanti and K. Tamvakis, Phys. Rev. D **68**, 024014, 2003 [hep-th/0303073]; S. Foffa, Phys. Rev. D **68**, 043511,2003 [hep-th/0304004]; J. L. Hovdebo and R. C. Myers, JCAP 0311:012,2003. [hep-th/0308088]; C.P. Burgess, F. Quevedo, R. Rabadan, G. Tasinato and I. Zavala, JCAP 0402:008,2004. [hep-th/0310122]; C. Germani, N.E. Grandi and A. Kehagias, Class. Quant. Grav.**25**, 135004,2008. [hep-th/0611246]; D. A. Easson, R. Gregory, G. Tasinato and I. Zavala, JHEP 0704:026,2007. [hep-th/0701252]; R. Maier, I. Damiao Soares and E.V. Tonini, Phys. Rev. D **79**, 023522,2009;

[23] M. A. Markov, Ann. Phys. (N.Y.) **155**, 333 (1984). V. Mukhanov and R. Brandenberger, Phys. Rev. Lett. **68**, 1969 (1992); R. Brandenberger, V. Mukhanov and A. Sornborger, Phys. Rev. D **48**, 1629 (1993).

[24] A. D. Linde, Phys. Lett. B **211**, 29 (1988).

[25] M.R. Setare, Phys. Lett. B **602**, 1, 2004 [e-Print: hep-th/0409055]; D.H. Coule, Class. Quant. Grav.**22**, R125, 2005 [e-Print: gr-qc/0412026]; F. T. Falciano, M. Lilley, P. Peter, Phys. Rev. D **77**, 083513, 2008. [e-Print: arXiv:0802.1196] H-H. Xiong, Y-F. Cai, T. Qiu, Y-S. Piao, X. Zhang, Phys. Lett. B **666**, 212, 2008. [e-Print: arXiv:0805.0413] M. V. Battisti, Phys. Rev. D **79**, 083506,2009. Yi Ling, Wei-Jia Li, Jian-Pin Wu, JCAP 0911:016,2009 [e-Print: arXiv:0909.4862]; X. Zhang, Phys. Lett. B **683**, 81,2010 [e-Print: arXiv:0909.4940]; R. Brandenberger, [e-Print: arXiv:0904.2835] Yi-Fu Cai and E.N. Saridakis, JCAP 0910:020,2009. [e-Print: arXiv:0906.1789]; Yi Ling and Q. Wu, Phys. Lett. B **687**, 103, 2010. [e-Print: arXiv:0811.2615]; C. Lin, R. H. Brandenberger and L. P. Levasseur [e-Print: arXiv:1007.2654] Yi-Fu Cai and E.N. Saridakis, Class. Quant. Grav.**28**, 035010,2011 [e-Print: arXiv:1007.3204]; Yi-Fu Cai, R. Brandenberger and X. Zhang, JCAP 1103:003,2011 [e-Print: arXiv:1101.0822]; K. Zhang, P. Wu and H. Yu, e-Print: arXiv:1202.1397;

[26] S. Nojiri and S. D. Odintsov, Phys.Rept. **505**, 59, 2011 [e-Print: arXiv:1011.0544]; T. Biswas *et al.*, Phys. Rev. Lett. **108**, 031101, 2012 [e-Print:arXiv:1110.5249]; A. S. Koshelev and S. Yu. Vernov, e-Print:arXiv:1202.1289.

[27] Ya. B. Zeldovich and I. D. Novikov, *The Structure and Evolution of the Universe*, University

of Chicago Press, Chicago, (1983).

[28] U. Alam, V. Sahni and A.A. Starobinsky, JCAP 04(2003)002.

[29] J. Frieman, C.T. Hill, A. Stebbins and I. Waga, Phys. Rev. Lett. **75**, 2077, 1995; K. Choi, Phys. Rev. D **62**, 043509 2000; R. Kallosh, A. Linde, S. Prokushkin and M. Shmakova, Phys. Rev. D **66**, 123503, 2002.

[30] J.D. Barrow, R. Bean and J. Magueijo, MNRAS **316**, L41, 2000; V. Sahni and Y. Shtanov, JCAP 0311 (2003) 014 [e-Print: astro-ph/0202346]; S. Nojiri and S D Odintsov, Phys. Lett. B **637**, 139, 2006 [e-Print: hep-th/0603062]; A. Shafieloo, V. Sahni and A. A. Starobinsky, Phys. Rev. D **80**, 101301, 2009, [e-Print: arXiv:0903.5141]; F.E.M. Costa and J.S. Alcaniz, Phys. Rev. D **81**, 043506, 2010, [e-Print: arXiv:0908.4251] J.C. Fabris, B. Fraga, N. Pinto-Neto and W. Zimdahl, JCAP 1004 (2010) 008 [e-Print: arXiv:0910.3246]; A. Shafieloo, V. Sahni and A. A. Starobinsky, Annalen Phys. **19**, 316, 2010; X. Chen, Y. Gong, E. N. Saridakis and Y. Gong, e-Print: arXiv:1111.6743; Yu.L. Bolotin, O.A. Lemets and D.A. Yerokhin, e-Print: arXiv:1108.0203; M. Li, X-D. Li, S. Wang and Y. Wang Commun.Theor.Phys. **56**, 525, 2011 [e-Print: arXiv:1103.5870]; M. Campista, B. Santos, J. Santos and J.S. Alcaniz, Phys. Lett. B **699**, 320, 2011 [e-Print: arXiv:1012.3943]; A.C.C. Guimaraes and J.A.S. Lima. Class.Quant.Grav. **28**, 125026, 2011, [e-Print: arXiv:1005.2986];

[31] M. G. Brown, K. Freese and W. H. Kinney, JCAP 0803:002,2008. [e-Print: astro-ph/0405353]

[32] A.A. Starobinsky, Sov. Astron. Lett., **4**, 82, 1978. D. Page, Class. Quant. Grav. **1**, 417, 1984. A.Yu. Kamenshchik, I.M. Khalatnikov and A.V. Toporensky, IJMP **D6**, 673, 1997. N.Cornish and E.P.S.Shellard, Phys. Rev. Lett. **81**, 3571, 1998.

[33] A. D. Linde, Phys. Lett. B **129**, 177, 1983

[34] V.A. Belinsky, L.P. Grishchuk, I.M. Khalatnikov and Ya.B. Zeldovich, Sov. Phys. JETP **63**, 195, 1985; Phys. Lett. B **155**, 232, 1985; V.A. Belinsky, *et al.*Prog. Theor. Phys. **79**, 676, 1988; G. Felder, L.A. Kofman and A. Linde, Phys. Rev. D **66**, 023507, 2002.

[35] Ya. B. Zeldovich, *My Universe: Selected Reviews*, edited by B. Ya. Zeldovich and M. V. Sazhin, Harwood Academic p. 93 (1992).

[36] V. Faraoni, arXiv:1201.1448.

[37] J. E. Lidsey and D. J. Mulryne, Phys. Rev. D **73**, 083508, 2006 [e-Print: hep-th/0601203];

[38] W. Hu, R. Barkana, A. Gruzinov, Phys. Rev. Lett. **85**, 1158 (2000)

[39] V. Sahni and L.M. Wang, Phys. Rev. D **62** 103517, 2000.

[40] T. Matos and L. Arturo Urena-Lopez, Phys. Rev. D **63**, 063506, 2001.

[41] G. Nicolis, *Introduction to Nonlinear Science*, Cambrige University Press, 1995.

[42] D.A. Dicus and W.W. Repko, Phys. Rev. D **67**, 083520, 2003 [e-Print: hep-ph/0211109]; W. Zimdahl, D.J. Schwarz, A.B. Balatkin and D. Pavon, Phys. Rev. D **64**, 063501, 2001 [e-Print: astro-ph/0009353]; S. Nojiri and S. D. Odintsov, Phys. Rev. D **72** 023003, 2005 [e-Print: hep-th/0505215]; J.C. Fabris, S.V.B. Goncalves and R. de Sa Ribeiro, Gen.Rel.Grav.38:495-506,2006 [e-Print: astro-ph/0503362]; I. Brevik and O. Gorbunova, Gen.Rel.Grav. **37**, 2039, 2005 [e-Print: gr-qc/0504001]; G. J. Mathews, N. Q. Lan and C. Kolda, Phys. Rev. D **78**, 043525, 2008 [e-Print: arXiv:0801.0853]; J-S Gagnon and J. Lesgourgues, JCAP 1109:026,2011 [e-Print: arXiv:1107.1503]

[43] T. Biswas and A. Mazumdar, Phys. Rev. D **80**, 023519, 2009. [e-Print: arXiv:0901.4930]

[44] B. Ratra, Phys. Rev. D **37**, 3406, 1988.

[45] S. Foster, *Scalar Field Cosmological Models With Hard Potential Walls*, [e-Print:gr-qc/9806113].

[46] Collins, C.B. and Hawking, S.W. Astroph. J. **180**, 317, 1973.

[47] Note that ambiguities exist in the notion of measure in general relativity as recently discussed in J. S. Schiffrin and R. M. Wald, *Measure and Probability in Cosmology*, e-Print:arXiv:1202.1637.

[48] Zeldovich, Ya.B. & Starobinsky A.A. (1971) *Zh. Eksp. Teor. Fiz.*, **61**, 2161. (*Sov. Physics JETP*, **34** (1972) 1159).

[49] R.M. Wald, Phys. Rev. D **28**, 2118, 1983; A.A. Starobinsky, Sov. Phys. JETP Lett. **37**, 66, 1983.

[50] I.G. Moss and V. Sahni, Phys. Lett. B **178**, 159, 1986.

[51] V. Sahni and P. Coles, Phys. Rep. **262**, 1, 1995.

[52] D. Wands, Phys. Rev. D **60**, 023507, 1999. F. Finelli and R. Brandenberger, Phys. Rev. D **65**, 103522, 2002 [e-Print:hep-th/0112249]; J. Martin and P. Peter, Phys. Rev. D **68**, 103517, 2003 [e-Print: hep-th/0307077]; T.J. Battefeld, S.P. Patil, R. Brandenberger, Phys. Rev. D **D70**, 066006, 2004 [e-Print:hep-th/0401010]; T.J. Battefeld and R. Brandenberger, Phys. Rev. D **D70**, 121302, 2004 [e-Print:hep-th/0406180]; T. J. Battefeld and G. Geshnizjani, Phys. Rev. D **73**, 064013 (2006) [hep-th/0503160]; T. J. Battefeld and G. Geshnizjani, Phys. Rev. D **73**, 048501 (2006) [hep-th/0506139]; F. C. Mena, D. J. Mulryne and R. Tavakol, Class.

Quant. Grav.**24**, 2721, 2007 [e-Print: gr-qc/0702064]; Y-S. Piao, [e-Print:arXiv:1012.2734]; S. D. P. Vitenti and N. Pinto-Neto, Phys. Rev. D **85**, 023524, 2012 [e-Print:arXiv:1111.0888]; R. Maier, S. Pereira, N. Pinto-Neto and B. B. Siffert, Phys. Rev. D **85**, 023508, 2012 [e-Print:arXiv:1111.0946]; Z-G. Liu and Y-S. Piao, e-Print:arXiv:1201.1371.

[53] G. Dvali and S.H.H. Tye, Phys. Lett. B **450**, 72, 1999; J. Garriga and V.F. Mukhanov, **458**, 219, 1999; R. Bean, D.J.H. Chung and G. Geshnizjani, Phys. Rev. D **78**, 023517, 2008.

[54] C. Armendariz-Picon, T. Damour and V.F. Mukhanov, Phys. Lett. B **458**, 209, 1999; C. Armendariz-Picon, V.F. Mukhanov and P.J. Steinhardt, Phys. Rev. Lett. **85**, 4438, 2000; Phys. Rev. D **63**, 103510, 2001.

[55] N. Arkani-Hamed, H.C. Cheng, M.A. Luty and S. Mukohyama, JHEP **0405**, 074, 2004.

[56] L. Amendola and S. Tsujikawa, *Dark Energy: Theory and Observations*, Cambridge University Press, 2010.

[57] C. Germani and A. Kehagias, Phys. Rev. Lett. **105**, 011302, 2010; T. Kobayashi, M. Yamaguchi and J. Yokoyama, Prog. Theor. Phys. **126**, 511, 2011; C. Deffayet, X. Giao, D. Steer and G. Zahariade, Phys. Rev. D **84**, 064039, 2011; C. Charmousis, E.J. Copeland, A. Padilla and P.M. Saffin, e-Print:arXiv:1106.2000; S. Tsujikawa, e-Print:arXiv:1201.5926.

[58] A.T. Mithani and A. Vilenkin, [e-Print:arXiv:1110.4096].